1 Essentials of Geometry

1.1 Identify Points, Lines, and Planes
1.2 Use Segments and Congruence
1.3 Use Midpoint and Distance Formulas
1.4 Measure and Classify Angles
1.5 Describe Angle Pair Relationships
1.6 Classify Polygons
1.7 Find Perimeter, Circumference, and Area

Before

In previous courses, you learned the following skills, which you'll use in Chapter 1: finding measures, evaluating expressions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK
Copy and complete the statement.
1. The distance around a rectangle is called its ___, and the distance around a circle is called its ___.
2. The number of square units covered by a figure is called its ___.

SKILLS AND ALGEBRA CHECK
Evaluate the expression. (Review p. 870 for 1.2, 1.3, 1.7.)
3. $|4 - 6|$  
4. $|3 - 11|$  
5. $|-4 + 5|$  
6. $|-8 - 10|$  

Evaluate the expression when $x = 2$. (Review p. 870 for 1.3–1.6.)
7. $5x$  
8. $20 - 8x$  
9. $-18 + 3x$  
10. $-5x - 4 + 2x$

Solve the equation. (Review p. 875 for 1.2–1.7.)
11. $274 = -2z$  
12. $8x + 12 = 60$  
13. $2y - 5 + 7y = -32$
14. $6p + 11 + 3p = -7$  
15. $8m - 5 = 25 - 2m$  
16. $-2n + 18 = 5n - 24$

@HomeTutor Prerequisite skills practice at classzone.com
In Chapter 1, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 59. You will also use the key vocabulary listed below.

**Big Ideas**

1. Describing geometric figures
2. Measuring geometric figures
3. Understanding equality and congruence

**Key Vocabulary**

- undefined terms, p. 2
  - point, line, plane
- defined terms, p. 3
- line segment, endpoints, p. 3
- ray, opposite rays, p. 3
- postulate, axiom, p. 9
- congruent segments, p. 11
- midpoint, p. 15
- segment bisector, p. 15
- acute, right, obtuse, straight angles, p. 25
- congruent angles, p. 26
- angle bisector, p. 28
- linear pair, p. 37
- vertical angles, p. 37
- polygon, p. 42
- convex, concave, p. 42
- n-gon, p. 43
- equilateral, equiangular, regular, p. 43

**Why?**

Geometric figures can be used to represent real-world situations. For example, you can show a climber’s position along a stretched rope by a point on a line segment.

**Animated Geometry**

The animation illustrated below for Exercise 35 on page 14 helps you answer this question: How far must a climber descend to reach the bottom of a cliff?

Your goal is to find the distance from a climber’s position to the bottom of a cliff.

Use the given information to enter a distance. Then check your answer.

**Other animations for Chapter 1:** pages 3, 21, 25, 43, and 52
1.1 Identify Points, Lines, and Planes

You studied basic concepts of geometry. You will name and sketch geometric figures. So you can use geometry terms in the real world, as in Ex. 13.

**Key Vocabulary**
- undefined terms: point, line, plane
- collinear points
- coplanar points
- defined terms
- line segment
- endpoints
- ray
- opposite rays
- intersection

In the diagram of a football field, the positions of players are represented by points. The yard lines suggest lines, and the flat surface of the playing field can be thought of as a plane.

In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

**KEY CONCEPT**

**Undefined Terms**

- **Point**: A point has no dimension. It is represented by a dot.

- **Line**: A line has one dimension. It is represented by a line with two arrowheads, but it extends without end.

  Through any two points, there is exactly one line. You can use any two points on a line to name it.

- **Plane**: A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

  Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.

**Collinear points** are points that lie on the same line. **Coplanar points** are points that lie in the same plane.
1.1 Identify Points, Lines, and Planes

**DEFINED TERMS** In geometry, terms that can be described using known words such as point or line are called defined terms.

**KEY CONCEPT** For Your Notebook

**Defined Terms: Segments and Rays**

Line \( AB \) (written as \( \overrightarrow{AB} \)) and points \( A \) and \( B \) are used here to define the terms below.

**Segment** The line segment \( AB \), or segment \( AB \), (written as \( \overrightarrow{AB} \)) consists of the endpoints \( A \) and \( B \) and all points on \( \overrightarrow{AB} \) that are between \( A \) and \( B \). Note that \( AB \) can also be named \( BA \).

**Ray** The ray \( AB \) (written as \( \overrightarrow{AB} \)) consists of the endpoint \( A \) and all points on \( \overrightarrow{AB} \) that lie on the same side of \( A \) as \( B \). Note that \( \overrightarrow{AB} \) and \( \overrightarrow{BA} \) are different rays.

If point \( C \) lies on \( \overrightarrow{AB} \) between \( A \) and \( B \), then \( \overrightarrow{CA} \) and \( \overrightarrow{CB} \) are opposite rays.

Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.
**Example 2** Name segments, rays, and opposite rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint $J$. Which of these rays are opposite rays?

**Solution**

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint $J$ are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint $J$ are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

**Guided Practice** for Example 2

Use the diagram in Example 2.

2. Give another name for $\overrightarrow{EF}$.

3. Are $\overrightarrow{HJ}$ and $\overrightarrow{JH}$ the same ray? Are $\overrightarrow{HJ}$ and $\overrightarrow{HG}$ the same ray? Explain.

**Intersections** Two or more geometric figures intersect if they have one or more points in common. The intersection of the figures is the set of points the figures have in common. Some examples of intersections are shown below.

**Example 3** Sketch intersections of lines and planes

a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

c. Sketch a plane and a line that intersects the plane at a point.

**Solution**

a. 

b. 

C.
EXAMPLE 4  Sketch intersections of planes

Sketch two planes that intersect in a line.

Solution

STEP 1  Draw a vertical plane. Shade the plane.

STEP 2  Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

STEP 3  Draw the line of intersection.

GUIDED PRACTICE  for Examples 3 and 4

4. Sketch two different lines that intersect a plane at the same point.

Use the diagram at the right.

5. Name the intersection of \( PQ \) and line \( k \).

6. Name the intersection of plane \( A \) and plane \( B \).

7. Name the intersection of line \( k \) and plane \( A \).

1.1 EXERCISES

1. VOCABULARY  Write in words what each of the following symbols means.

   a. \( Q \)  
   b. \( MN \)  
   c. \( ST \)  
   d. \( FG \)

2. WRITING  Compare collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? Explain.

NAMING POINTS, LINES, AND PLANES  In Exercises 3–7, use the diagram.

3. Give two other names for \( \overrightarrow{WQ} \).

4. Give another name for plane \( V \).

5. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.

6. Name a point that is not coplanar with points \( R, S, \) and \( T \).

7. WRITING  Is point \( W \) coplanar with points \( Q \) and \( R \)? Explain.
NAMING SEGMENTS AND RAYS  In Exercises 8–12, use the diagram.

8. What is another name for $\overrightarrow{ZY}$?

9. Name all rays with endpoint $V$.

10. Name two pairs of opposite rays.

11. Give another name for $\overrightarrow{WV}$.

12. ERROR ANALYSIS A student says that $\overrightarrow{VW}$ and $\overrightarrow{VZ}$ are opposite rays because they have the same endpoint. Describe the error.

13. TAKS REASONING Which statement about the diagram at the right is true?

   A. $A$, $B$, and $C$ are collinear.
   B. $C$, $D$, $E$, and $G$ are coplanar.
   C. $B$ lies on $\overrightarrow{GE}$.
   D. $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are opposite rays.

SKETCHING INTERSECTIONS Sketch the figure described.

14. Three lines that lie in a plane and intersect at one point

15. One line that lies in a plane, and one line that does not lie in the plane

16. TAKS REASONING Line $AB$ and line $CD$ intersect at point $E$. Which of the following are opposite rays?

   A. $\overrightarrow{EC}$ and $\overrightarrow{ED}$
   B. $\overrightarrow{CE}$ and $\overrightarrow{DE}$
   C. $\overrightarrow{AB}$ and $\overrightarrow{BA}$
   D. $\overrightarrow{AE}$ and $\overrightarrow{BE}$

READING DIAGRAMS In Exercises 17–22, use the diagram at the right.

17. Name the intersection of $\overrightarrow{PR}$ and $\overrightarrow{HR}$.

18. Name the intersection of plane $EFG$ and plane $FGS$.

19. Name the intersection of plane $PQS$ and plane $HGS$.

20. Are points $P$, $Q$, and $F$ collinear? Are they coplanar?

21. Are points $P$ and $G$ collinear? Are they coplanar?

22. Name three planes that intersect at point $E$.

23. SKETCHING PLANES Sketch plane $J$ intersecting plane $K$. Then draw a line $l$ on plane $J$ that intersects plane $K$ at a single point.

24. NAMING RAYS Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.

25. SKETCHING Draw three noncollinear points $J$, $K$, and $L$. Sketch $\overrightarrow{JK}$ and add a point $M$ on $\overrightarrow{JK}$. Then sketch $\overrightarrow{ML}$.

26. SKETCHING Draw two points $P$ and $Q$. Then sketch $\overrightarrow{PQ}$. Add a point $R$ on the ray so that $Q$ is between $P$ and $R$. 

\[ = \text{WORKED-OUT SOLUTIONS on p. WS1} \]

\[ = \text{TAKS PRACTICE AND REASONING} \]
**ALGEBRA** In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. \( y = x - 4; \ A(5, 1) \)
28. \( y = x + 1; \ A(1, 0) \)
29. \( y = 3x + 4; \ A(7, 1) \)
30. \( y = 4x + 2; \ A(1, 6) \)
31. \( y = 3x - 2; \ A(-1, -5) \)
32. \( y = -2x + 8; \ A(-4, 0) \)

**GRAPHING** Graph the inequality on a number line. Tell whether the graph is a segment, a ray or rays, a point, or a line.

33. \( x \leq 3 \)
34. \( x \geq -4 \)
35. \(-7 \leq x \leq 4 \)
36. \( x \geq 5 \) or \( x \leq -2 \)
37. \( x \geq -1 \) or \( x \leq 5 \)
38. \( |x| \leq 0 \)

**CHALLENGE** Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch.

a. None of the three planes intersect.

b. The three planes intersect in one line.

c. The three planes intersect in one point.

d. Two planes do not intersect. The third plane intersects the other two.

e. Exactly two planes intersect. The third plane does not intersect the other two.

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**EVERYDAY INTERSECTIONS** What kind of geometric intersection does the photograph suggest?

40. [Image of a table]
41. [Image of a table]
42. [Image of a table]

43. **TAKS REASONING** Explain why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

44. **SURVEYING** A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.

a. When the tripod is sitting on a level surface, are the tips of the legs coplanar?

b. Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? Explain.
45. **MULTI-STEP PROBLEM** In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with two vanishing points.

![Diagram of a house with two vanishing points.](image)

- a. Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W. Join points E and F to the vanishing point V.
- b. Label the intersection of \( \overrightarrow{EV} \) and \( \overrightarrow{AW} \) as G. Label the intersection of \( \overrightarrow{FV} \) and \( \overrightarrow{BW} \) as H.
- c. Using heavy dashed lines, draw the hidden edges of the house: \( \overrightarrow{AG}, \overrightarrow{EG}, \overrightarrow{BH}, \overrightarrow{FH}, \) and \( \overrightarrow{GH} \).

46. **CHALLENGE** Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.

![Images of 2, 3, and 4 streets.](images)

- a. A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in the town? 6 streets?
- b. *Describe a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town.*

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**Mixed Review for TAKS**

47. **TAKS PRACTICE** Which set of coordinates describes a function?

- **TAKS Obj. 1**
  - A. \( \{(2, 2), (4, 0), (6, 0), (2, 0)\} \)
  - B. \( \{(0, 3), (1, 0), (1, -1), (3, -3)\} \)
  - C. \( \{(4, 0), (-2, 2), (0, 2), (-2, 4)\} \)
  - D. \( \{(3, -1), (1, -1), (-3, 0), (-1, 3)\} \)

48. **TAKS PRACTICE** How many blocks are visible in the top view of the figure at the right? **TAKS Obj. 7**

- A. 3 blocks
- B. 4 blocks
- C. 5 blocks
- D. 6 blocks
1.2 Use Segments and Congruence

**Key Vocabulary**
- postulate, axiom
- coordinate
- distance
- between
- congruent segments

You learned about points, lines, and planes.
Now you will use segment postulates to identify congruent segments.
So you can calculate flight distances, as in Ex. 33.

In Geometry, a rule that is accepted without proof is called a postulate or axiom. A rule that can be proved is called a theorem, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

**POSTULATE 1 Ruler Postulate**

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.

The distance between points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

In the diagrams above, the small numbers in the coordinates $x_1$ and $x_2$ are called subscripts. The coordinates are read as “$x$ sub one” and “$x$ sub two.”

The distance between points $A$ and $B$, or $AB$, is also called the length of $AB$.

**EXAMPLE 1 Apply the Ruler Postulate**

Measure the length of $ST$ to the nearest tenth of a centimeter.

**Solution**

Align one mark of a metric ruler with $S$. Then estimate the coordinate of $T$. For example, if you align $S$ with 2, $T$ appears to align with 5.4.

$ST = |5.4 - 2| = 3.4$ Use Ruler Postulate.

The length of $ST$ is about 3.4 centimeters.
**Example 2**  Apply the Segment Addition Postulate

**MAPS** The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.

**Solution**

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

\[ LS = LT + TS = 380 + 360 = 740 \]

- The distance from Lubbock to St. Louis is about 740 miles.

**Guided Practice** for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest \( \frac{1}{8} \) inch.

1. \( MN \)

2. \( PQ \)

In Exercises 3 and 4, use the diagram shown.

3. Use the Segment Addition Postulate to find \( XZ \).

4. In the diagram, \( WY = 30 \). Can you use the Segment Addition Postulate to find the distance between points \( W \) and \( Z \)? Explain your reasoning.
1.2 Use Segments and Congruence

CONGRUENT SEGMENTS Line segments that have the same length are called congruent segments. In the diagram below, you can say “the length of }\overline{AB}\text{ is equal to the length of }\overline{CD}," or you can say “}\overline{AB}\text{ is congruent to }\overline{CD}.” The symbol }\equiv\text{ means “is congruent to.”

EXAMPLE 3 Find a length

Use the diagram to find }\overline{GH}.

Solution

Use the Segment Addition Postulate to write an equation. Then solve the equation to find }\overline{GH}.

\[
\begin{align*}
\text{ }\overline{FH} &= \text{ }\overline{FG} + \text{ }\overline{GH} & \text{ Segment Addition Postulate} \\
36 &= 21 + \text{ }\overline{GH} & \text{ Substitute 36 for }\overline{FH} \text{ and 21 for }\overline{FG}. \\
15 &= \text{ }\overline{GH} & \text{ Subtract 21 from each side.}
\end{align*}
\]

EXAMPLE 4 Compare segments for congruence

Plot }\overline{J(−3, 4)}, \overline{K(2, 4)}, \overline{L(1, 3)}, \text{ and }\overline{M(1, −2)}\text{ in a coordinate plane. Then determine whether }\overline{JK}\text{ and }\overline{LM}\text{ are congruent.}

Solution

To find the length of a horizontal segment, find the absolute value of the difference of the }x\text{-coordinates of the endpoints.}

\[
\text{ }\overline{JK} = |2 − (−3)| = 5 \quad \text{ Use Ruler Postulate.}
\]

To find the length of a vertical segment, find the absolute value of the difference of the }y\text{-coordinates of the endpoints.}

\[
\text{ }\overline{LM} = |−2 − 3| = 5 \quad \text{ Use Ruler Postulate.}
\]

}\overline{JK}\text{ and }\overline{LM}\text{ have the same length. So, }\overline{JK} \equiv \overline{LM}.

GUIDED PRACTICE for Examples 3 and 4

5. Use the diagram at the right to find }\overline{WX}.

6. Plot the points }\overline{A(−2, 4)}, \overline{B(3, 4)}, \overline{C(0, 2)}, \text{ and }\overline{D(0, −2)}\text{ in a coordinate plane. Then determine whether }\overline{AB}\text{ and }\overline{CD}\text{ are congruent.
In Exercises 1 and 2, use the diagram at the right.

1. **VOCABULARY**  Explain what $\overline{MN}$ means and what $\overline{PM}$ means.

2. **WRITING**  Explain how you can find $\overline{PN}$ if you know $\overline{PQ}$ and $\overline{QN}$. How can you find $\overline{PN}$ if you know $\overline{MP}$ and $\overline{MN}$?

**MEASUREMENT**  Measure the length of the segment to the nearest tenth of a centimeter.

3. $\overline{AB}$
4. $\overline{CD}$
5. $\overline{EF}$

**SEGMENT ADDITION POSTULATE**  Find the indicated length.

6. Find $\overline{MP}$.
7. Find $\overline{RT}$.
8. Find $\overline{UW}$.

9. Find $\overline{XY}$.
10. Find $\overline{BC}$.
11. Find $\overline{DE}$.

12. **ERROR ANALYSIS**  In the figure at the right, $\overline{AC} = 14$ and $\overline{AB} = 9$. Describe and correct the error made in finding $\overline{BC}$.

**CONGRUENCE**  In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

13. $A(0, 1), B(4, 1), C(1, 2), D(1, 6); \overline{AB}$ and $\overline{CD}$
14. $J(-6, -8), K(-6, 2), L(-2, -4), M(-6, -4); \overline{JK}$ and $\overline{LM}$
15. $R(-200, 300), S(200, 300), T(300, -200), U(300, 100); \overline{RS}$ and $\overline{TU}$

**ALGEBRA**  Use the number line to find the indicated distance.

16. $\overline{JK}$
17. $\overline{JL}$
18. $\overline{JM}$
19. $\overline{KM}$

20. **TAKS REASONING**  Use the diagram. Is it possible to use the Segment Addition Postulate to show that $\overline{FB} > \overline{CB}$ or that $\overline{AC} > \overline{DB}$? Explain.
1.2 Use Segments and Congruence

**FINDING LENGTHS** In the diagram, points V, W, X, Y, and Z are collinear, VZ = 52, XZ = 20, and WX = XY = YZ. Find the indicated length.

21. WX  
22. VW  
23. WY  
24. VX  
25. WZ  
26. VY

27. **TAKS REASONING** Use the diagram.

What is the length of EG?

A. 1  
B. 4.4  
C. 10  
D. 16

28. **ALGEBRA** Point S is between R and T on RT. Use the given information to write an equation in terms of x. Solve the equation. Then find RS and ST.

29. RS = 3x - 16
   ST = 4x - 8
   RT = 21

30. RS = 2x - 8
   ST = 3x - 10
   RT = 17

31. **CHALLENGE** In the diagram, AB ≅ BC, AC ≅ CD, and AD = 12. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? Explain.

32. **SCIENCE** The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest \( \frac{1}{4} \) inch. About how much longer is the walkingstick’s abdomen than its thorax?

33. **MODEL AIRPLANE** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane’s position at three different points during its flight.

a. Find the total distance the model airplane flew.

b. The model airplane’s flight lasted nearly 38 hours. Estimate the airplane’s average speed in miles per hour.
34. **TAKS REASONING** The bar graph shows the win-loss record for a lacrosse team over a period of three years.

a. Use the scale to find the length of the yellow bar for each year. What does the length represent?

b. For each year, find the percent of games lost by the team.

c. Explain how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.

35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let $A$ represent the point where the rope is secured at the top of the cliff, let $B$ represent the climber’s position, and let $C$ represent the point where the rope is secured at the bottom of the cliff.

a. **Model** Draw and label a line segment that represents the situation.

b. **Calculate** If $AC$ is 52 feet and $AB$ is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

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37. **TAKS PRACTICE** Which function best describes the graph at the right? **TAKS Obj. 3**

- A) $f(x) = -\frac{2}{3}x + 2$
- B) $f(x) = -\frac{3}{2}x - 2$
- C) $f(x) = \frac{2}{3}x + 2$
- D) $f(x) = \frac{3}{2}x - 2$

38. **TAKS PRACTICE** Which coordinates represent a point that lies in Quadrant II? **TAKS Obj. 6**

- F) $(1, 5)$
- G) $(-1, 5)$
- H) $(1, -5)$
- I) $(-1, -5)$
1.3 Use Midpoint and Distance Formulas

**Before**
You found lengths of segments.

**Now**
You will find lengths of segments in the coordinate plane.

**Why?**
So you can find an unknown length, as in Example 1.

**Key Vocabulary**
- midpoint
- segment bisector

**ACTIVITY  FOLD A SEGMENT BISECTOR**

**STEP 1**  
Draw $\overline{AB}$ on a piece of paper.

**STEP 2**  
Fold the paper so that $B$ is on top of $A$.

**STEP 3**  
Label point $M$. Compare $AM$, $MB$, and $AB$.

**MIDPOINTS AND BISECTORS**  
The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector **bisects** a segment.

$M$ is the midpoint of $\overline{AB}$.  
So, $AM = MB$ and $AM = MB$.

$\overline{CD}$ is a segment bisector of $\overline{AB}$.  
So, $AM = MB$ and $AM = MB$.

**EXAMPLE 1  Find segment lengths**

**SKATEBOARD**  
In the skateboard design, $\overline{VW}$ **bisects** $\overline{XY}$ at point $T$, and $XT = 39.9$ cm. Find $XY$.

**Solution**
Point $T$ is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

$XY = XT + TY$  
= $39.9 + 39.9$  
= 79.8 cm

**Segment Addition Postulate**  
**Substitute.**  
**Add.**
**Example 2** Use algebra with segment lengths

**ALGEBRA** Point \( M \) is the midpoint of \( VW \). Find the length of \( VM \).

**Solution**

**Step 1** Write and solve an equation. Use the fact that \( VM = MW \).

\[
VM = MW \quad \text{Write equation.}
\]

\[
4x - 1 = 3x + 3 \quad \text{Substitute.}
\]

\[
x - 1 = 3 \quad \text{Subtract 3x from each side.}
\]

\[
x = 4 \quad \text{Add 1 to each side.}
\]

**Step 2** Evaluate the expression for \( VM \) when \( x = 4 \).

\[
VM = 4x - 1 = 4(4) - 1 = 15
\]

So, the length of \( VM \) is 15.

**Check** Because \( VM = MW \), the length of \( MW \) should be 15. If you evaluate the expression for \( MW \), you should find that \( MW = 15 \).

\[
MW = 3x + 3 = 3(4) + 3 = 15
\]

✓

**Guided Practice** for Examples 1 and 2

In Exercises 1 and 2, identify the segment bisector of \( PQ \). Then find \( PQ \).

1. 

2.

**Coordinate Plane** You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

**Key Concept**

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the \( x \)-coordinates and of the \( y \)-coordinates of the endpoints.

If \( A(x_1, y_1) \) and \( B(x_2, y_2) \) are points in a coordinate plane, then the midpoint \( M \) of \( AB \) has coordinates

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
**EXAMPLE 3** Use the Midpoint Formula

a. **FIND MIDPOINT** The endpoints of $RS$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint $M$.

b. **FIND ENDPOINT** The midpoint of $JK$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint $K$.

**Solution**

a. **FIND MIDPOINT** Use the Midpoint Formula.

$$M\left(\frac{1 + 4}{2}, \frac{-3 + 2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

The coordinates of the midpoint $M$ are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.

b. **FIND ENDPOINT** Let $(x, y)$ be the coordinates of endpoint $K$. Use the Midpoint Formula.

**STEP 1** Find $x$.

**STEP 2** Find $y$.

$$1 + x = 2 \quad \quad \quad 4 + y = 1$$

$$1 + x = 4 \quad \quad \quad 4 + y = 2$$

$$x = 3 \quad \quad \quad y = -2$$

The coordinates of endpoint $K$ are $(3, -2)$.

**GUIDED PRACTICE** for Example 3

3. The endpoints of $AB$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint $M$.

4. The midpoint of $VW$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint $V$.

**DISTANCE FORMULA** The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

**KEY CONCEPT**

**The Distance Formula**

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between $A$ and $B$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
The Distance Formula is based on the Pythagorean Theorem, which you will see again when you work with right triangles in Chapter 7.

**Distance Formula**

\[(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2\]

**Pythagorean Theorem**

\[c^2 = a^2 + b^2\]

---

**Example 4 TAKS Practice: Multiple Choice**

What is the approximate length of \(RS\), with endpoints \(R(3, 1)\) and \(S(\text{-}1, \text{-}5)\)?

- **A** 1.6 units
- **B** 3.2 units
- **C** 4.8 units
- **D** 7.2 units

**Solution**

\[RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

Substitute.

\[= \sqrt{(-1 - 3)^2 + ((-5) - 1)^2}\]

Subtract.

\[= \sqrt{(-4)^2 + (-6)^2}\]

Evaluate powers.

\[= \sqrt{16 + 36}\]

Add.

\[= \sqrt{52}\]

Use a calculator to approximate the square root.

\[\approx 7.21\]

The correct answer is **D**.  \(\text{A B C D}\)

---

**Guided Practice** for Example 4

5. In Example 4, does it matter which ordered pair you choose to substitute for \((x_1, y_1)\) and which ordered pair you choose to substitute for \((x_2, y_2)\)? Explain.

6. What is the approximate length of \(AB\), with endpoints \(A(-3, 2)\) and \(B(1, -4)\)?

- **A** 6.1 units
- **B** 7.2 units
- **C** 8.5 units
- **D** 10.0 units
1.3 EXERCISES

1. **VOCABULARY** Copy and complete: To find the length of \(AB\), with endpoints \(A(-7, 5)\) and \(B(4, -6)\), you can use the __________.

2. **WRITING** Explain what it means to bisect a segment. Why is it impossible to bisect a line?

**FINDING LENGTHS** Line \(l\) bisects the segment. Find the indicated length.

3. Find \(RT\) if \(RS = 5\frac{1}{8}\) in.
4. Find \(UW\) if \(VW = 5\frac{3}{8}\) in.
5. Find \(EG\) if \(EF = 13\) cm.
6. Find \(BC\) if \(AC = 19\) cm.
7. Find \(QR\) if \(PR = 9\frac{1}{2}\) in.
8. Find \(LM\) if \(LN = 137\) mm.

9. **SEGMENT BISECTOR** Line \(RS\) bisects \(PQ\) at point \(R\). Find \(RQ\) if \(PQ = 4\frac{3}{4}\) inches.

10. **SEGMENT BISECTOR** Point \(T\) bisects \(UV\). Find \(UV\) if \(UT = 2\frac{7}{8}\) inches.

**ALGEBRA** In each diagram, \(M\) is the midpoint of the segment. Find the indicated length.

11. Find \(AM\).
12. Find \(EM\).
13. Find \(JM\).
14. Find \(PR\).
15. Find \(SU\).
16. Find \(XZ\).

**FINDING MIDPOINTS** Find the coordinates of the midpoint of the segment with the given endpoints.

17. \(C(3, 5)\) and \(D(7, 5)\)
18. \(E(0, 4)\) and \(F(4, 3)\)
19. \(G(-4, 4)\) and \(H(6, 4)\)
20. \(J(-7, -5)\) and \(K(-3, 7)\)
21. \(P(-8, -7)\) and \(Q(11, 5)\)
22. \(S(-3, 3)\) and \(T(-8, 6)\)

23. **WRITING** Develop a formula for finding the midpoint of a segment with endpoints \(A(0, 0)\) and \(B(m, n)\). Explain your thinking.
24. **ERROR ANALYSIS** *Describe* the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.

**EXAMPLE 4**

on p. 18
for Exs. 31–34

**FINDING ENDPOINTS** *Use the given endpoint $R$ and midpoint $M$ of $\overline{RS}$ to find the coordinates of the other endpoint $S$.**

25. $R(3, 0), M(0, 5)$
26. $R(5, 1), M(1, 4)$
27. $R(6, -2), M(5, 3)$
28. $R(-7, 11), M(2, 1)$
29. $R(4, 2), M(2, 8)$
30. $R(-4, -6), M(3, -4)$

**DISTANCE FORMULA** *Find the length of the segment. Round to the nearest tenth of a unit.*

31. $A(5, 4), P(1, 2)$
32. $A(-3, 5), R(2, 3)$
33. $S(-1, 2), T(3, -2)$

34. **TAKS REASONING** The endpoints of $\overline{MN}$ are $M(-3, -9)$ and $N(4, 8)$.
What is the approximate length of $\overline{MN}$?

- **A** 1.4 units
- **B** 7.2 units
- **C** 13 units
- **D** 18.4 units

**NUMBER LINE** *Find the length of the segment. Then find the coordinate of the midpoint of the segment.*

35. $I$ no solution
36. $I$ no solution
37. $I$ no solution

38. $I$ no solution
39. $I$ no solution
40. $I$ no solution

41. **TAKS REASONING** The endpoints of $\overline{LF}$ are $L(-2, 2)$ and $F(3, 1)$.
The endpoints of $\overline{JR}$ are $J(1, -1)$ and $R(-2, -3)$. What is the approximate difference in the lengths of the two segments?

- **A** 2.24
- **B** 2.86
- **C** 5.10
- **D** 7.96

42. **TAKS REASONING** One endpoint of $\overline{PQ}$ is $P(-2, 4)$. The midpoint of $\overline{PQ}$ is $M(1, 0)$. *Explain how to find $PQ$.***

**COMPARING LENGTHS** *The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.*

43. $\overline{AB}: A(0, 2), B(-3, 8)$
44. $\overline{EF}: E(1, 4), F(5, 1)$
45. $\overline{JK}: J(-4, 0), K(4, 8)$

\[ \overline{CD}: C(-2, 2), D(0, -4) \]
\[ \overline{GH}: G(-3, 1), H(1, 6) \]
\[ \overline{LM}: L(-4, 2), M(3, -7) \]

46. **ALGEBRA** Points $S$, $T$, and $P$ lie on a number line. Their coordinates are 0, 1, and $x$, respectively. Given $SP = PT$, what is the value of $x$?

47. **CHALLENGE** $M$ is the midpoint of $\overline{JK}$, $JM = \frac{x}{8}$, and $JK = \frac{3x}{4} - 6$. Find $MK$. 

= **WORKED-OUT SOLUTIONS** on p. WS1

= **TAKS PRACTICE AND REASONING**
48. **WINDMILL** In the photograph of a windmill, \( \overline{ST} \) bisects \( \overline{QR} \) at point \( M \). The length of \( \overline{QM} \) is 18\( \frac{1}{2} \) feet. Find \( QR \) and \( MR \).

49. **DISTANCES** A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

**ARCHAEOLOGY** The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.

50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.
   a. \( A \) and \( B \)
   b. \( B \) and \( C \)
   c. \( C \) and \( D \)
   d. \( A \) and \( D \)
   e. \( B \) and \( D \)
   f. \( A \) and \( C \)

51. Which two objects are closest to each other? Which two are farthest apart?

52. **WATER POLO** The diagram shows the positions of three players during part of a water polo match. Player \( A \) throws the ball to Player \( B \), who then throws it to Player \( C \). How far did Player \( A \) throw the ball? How far did Player \( B \) throw the ball? How far would Player \( A \) have thrown the ball if he had thrown it directly to Player \( C \)? Round all answers to the nearest tenth of a meter.
53. **TAKS REASONING** As shown, a path goes around a triangular park.

a. Find the distance around the park to the nearest yard.

b. A new path and a bridge are constructed from point \( Q \) to the midpoint \( M \) of \( PR \). Find \( QM \) to the nearest yard.

c. A man jogs from \( P \) to \( Q \) to \( M \) to \( R \) to \( Q \) and back to \( P \) at an average speed of 150 yards per minute. About how many minutes does it take? **Explain**.

54. **CHALLENGE** \( AB \) bisects \( CD \) at point \( M \), \( CD \) bisects \( AB \) at point \( M \), and \( AB = 4 \cdot CM \). Describe the relationship between \( AM \) and \( CD \).

---

**Mixed Review for TAKS**

55. **TAKS PRACTICE** What are the solutions to \( x^2 + 4x = 5 \)? **TAKS** Obj. 5

- **A** \( x = 1 \) and \( x = 5 \)
- **B** \( x = 1 \) and \( x = -5 \)
- **C** \( x = -1 \) and \( x = 5 \)
- **D** \( x = -1 \) and \( x = -5 \)

56. **TAKS PRACTICE** Juan is putting up a wallpaper border along the top of each wall in his rectangular living room. The border costs $9.25 per roll plus 7.75% sales tax. One roll is 15 feet long. What other information is needed to determine the number of rolls of border he needs to purchase? **TAKS** Obj. 10

- **F** The perimeter of the room
- **G** The total cost of each roll of border
- **H** The weight of one roll of border
- **J** The height of the walls in the room

57. **TAKS PRACTICE** Sally can jog at a rate of 6.5 miles per hour. If she continued in a straight path at this rate, what distance would she travel in 24 minutes? **TAKS** Obj. 9

- **A** 2.2 miles
- **B** 2.4 miles
- **C** 2.6 miles
- **D** 2.8 miles

---

**Quiz for Lessons 1.1–1.3**

1. Sketch two lines that intersect the same plane at two different points. The lines intersect each other at a point not in the plane. (p. 2)

In the diagram of collinear points, \( AE = 26 \), \( AD = 15 \), and \( AB = BC = CD \). Find the indicated length. (p. 9)

- **2** \( DE \)
- **3** \( AB \)
- **4** \( AC \)
- **5** \( BD \)
- **6** \( CE \)
- **7** \( BE \)

8. The endpoints of \( RS \) are \( R(-2, -1) \) and \( S(2, 3) \). Find the coordinates of the midpoint of \( RS \). Then find the distance between \( R \) and \( S \). (p. 15)
1. **MULTI-STEP PROBLEM** All tickets for a.

2. **WALL FRAME** The diagram shows the frame for a wall. $FH$ represents a vertical board and $EG$ represents a brace. The brace bisects $FH$. How long is $FG$? **TEKS G.7.C**

3. **COORDINATE PLANE** Point $E$ is the midpoint of $AB$ and $CD$. The coordinates of $A$, $B$, and $C$ are $A(-4, 5)$, $B(6, -5)$, and $C(2, 8)$. What are the coordinates of point $D$? **TEKS G.7.A**

4. **NEW ROAD** The diagram shows existing roads and a planned new road, represented by $CE$. About how much shorter is a trip from $B$ to $F$, where possible, using the new road instead of the existing roads? **TEKS G.7.C**

5. **TRAVELING SALESPERSON** Jill is a salesperson who needs to visit Towns $A$, $B$, and $C$. On the map, $AB = 18.7$ km and $BC = 2AB$. Starting at Town $A$, Jill travels along the road shown to Town $B$, then solve to Town $C$, and returns to Town $A$. What distance does Jill travel? **TEKS G.7.C**

6. **CLOCK** In the photo of the clock below, which segment represents the intersection of planes $ABC$ and $BFE$? **TEKS G.6.C**

7. **MIDPOINT FORMULA** Point $M$ is the midpoint of $PQ$. $PM = (23x + 5)$ inches and $MQ = (25x - 4)$ inches. Find the length (in inches) of $PQ$. **TEKS G.7.C**

8. **HIKING TRAIL** Tom is hiking on a trail that lies along a straight railroad track. The total length of the trail is 5.4 kilometers. Starting from the beginning of the trail, he has been walking for 45 minutes at an average speed of 2.4 kilometers per hour. What is the length (in kilometers) to the end of the trail? **TEKS G.7.C**
1.4 Measure and Classify Angles

You named and measured line segments. You will name, measure, and classify angles. So you can identify congruent angles, as in Example 4.

Key Vocabulary
• angle
  • acute, right, obtuse, straight
• sides, vertex of an angle
• measure of an angle
• congruent angles
• angle bisector

An angle consists of two different rays with the same endpoint. The rays are the sides of the angle. The endpoint is the vertex of the angle.

The angle with sides \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) can be named \( \angle BAC \), \( \angle CAB \), or \( \angle A \). Point \( A \) is the vertex of the angle.

**Example 1** Name angles

Name the three angles in the diagram.

\( \angle WXY, \text{ or } \angle YWX \)

\( \angle YXZ, \text{ or } \angle ZXY \)

\( \angle WXZ, \text{ or } \angle ZWX \)

You should not name any of these angles \( \angle X \) because all three angles have \( X \) as their vertex.

**Measuring Angles** A protractor can be used to approximate the measure of an angle. An angle is measured in units called degrees (°). For instance, the measure of \( \angle WXZ \) in Example 1 above is 32°. You can write this statement in two ways.

**Words** The measure of \( \angle WXZ \) is 32°.

**Symbols** \( m\angle WXZ = 32° \)

**Postulate 3** Protractor Postulate

Consider \( \overrightarrow{OB} \) and a point \( A \) on one side of \( \overrightarrow{OB} \).

The rays of the form \( \overrightarrow{OA} \) can be matched one to one with the real numbers from 0 to 180.

The measure of \( \angle AOB \) is equal to the absolute value of the difference between the real numbers for \( \overrightarrow{OA} \) and \( \overrightarrow{OB} \).
CLASSIFYING ANGLES Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

![Diagram of acute, right, obtuse, and straight angles]

**GUIDED PRACTICE** for Examples 1 and 2

1. Name all the angles in the diagram at the right. Which angle is a right angle?
2. Draw a pair of opposite rays. What type of angle do the rays form?

**POSTULATE 4 Angle Addition Postulate**

**Words** If $P$ is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

**Symbols** If $P$ is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$. 

---

**Corrected Answer**: For the guided practice, the angles in the diagram are $\angle PHQ$, $\angle HQR$, $\angle LPS$, $\angle PQS$, $\angle QPS$, and $\angle QRS$. The right angle is $\angle QPS$. Drawing a pair of opposite rays would create two adjacent angles, such as $\angle PQS$ and $\angle QPS$, which would be a right angle.
**Example 3** Find angle measures

**ALGEBRA** Given that $m\angle LKN = 145^\circ$, find $m\angle LKM$ and $m\angle MKN$.

**Solution**

**STEP 1** Write and solve an equation to find the value of $x$.

\[
m\angle LKN = m\angle LKM + m\angle MKN \quad \text{Angle Addition Postulate}
\]

\[
145^\circ = (2x + 10)^\circ + (4x - 3)^\circ \quad \text{Substitute angle measures.}
\]

\[
145 = 6x + 7
\]

\[
138 = 6x
\]

\[
x = \frac{138}{6} = 23
\]

**STEP 2** Evaluate the given expressions when $x = 23$.

\[
m\angle LKM = (2x + 10)^\circ = (2 \cdot 23 + 10)^\circ = 56^\circ
\]

\[
m\angle MKN = (4x - 3)^\circ = (4 \cdot 23 - 3)^\circ = 89^\circ
\]

So, $m\angle LKM = 56^\circ$ and $m\angle MKN = 89^\circ$.

**Guided Practice** for Example 3

Find the indicated angle measures.

3. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.

4. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.

**Congruent Angles** Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that “the measure of angle $A$ is equal to the measure of angle $B$,” or you can say “angle $A$ is congruent to angle $B$.”

**Read Diagrams** Matching arcs are used to show that angles are congruent. If more than one pair of angles are congruent, double arcs are used, as in Example 4 on page 27.
**Example 4** Identify congruent angles

**Trapeze** The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If \( m\angle DEG = 157^\circ \), what is \( m\angle GKL \)?

**Solution**

There are two pairs of congruent angles:

\[ \angle DEF \cong \angle KJL \text{ and } \angle DEG \cong \angle GKL. \]

Because \( \angle DEG \cong \angle GKL \), \( m\angle DEG = m\angle GKL \). So, \( m\angle GKL = 157^\circ \).

---

**Guided Practice** for Example 4

Use the diagram shown at the right.

5. Identify all pairs of congruent angles in the diagram.

6. In the diagram, \( m\angle PQR = 130^\circ \), \( m\angle QRS = 84^\circ \), and \( m\angle TSR = 121^\circ \). Find the other angle measures in the diagram.

---

**Activity** Fold an Angle Bisector

**Step 1**

Use a straightedge to draw and label an acute angle, \( \angle ABC \).

**Step 2**

Fold the paper so that \( \overline{BC} \) is on top of \( \overline{BA} \).

**Step 3**

Draw a point \( D \) on the fold inside \( \angle ABC \). Then measure \( \angle ABD \), \( \angle DBC \), and \( \angle ABC \). What do you observe?
An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on page 27, \( \overrightarrow{BD} \) bisects \( \angle ABC \). So, \( \angle ABD \cong \angle DBC \) and \( m\angle ABD = m\angle DBC \).

### Example 5  Double an angle measure

In the diagram at the right, \( \overrightarrow{YW} \) bisects \( \angle XYZ \), and \( m\angle XYW = 18^\circ \). Find \( m\angle XYZ \).

**Solution**

By the Angle Addition Postulate, \( m\angle XYZ = m\angle XYW + m\angle WYZ \). Because \( \overrightarrow{YW} \) bisects \( \angle XYZ \), you know that \( \angle XYW \cong \angle WYZ \).

So, \( m\angle XYW = m\angle WYZ \), and you can write

\[
m\angle XYZ = m\angle XYW + m\angle WYZ = 18^\circ + 18^\circ = 36^\circ.
\]

### Guided Practice for Example 5

7. Angle \( \angle MNP \) is a straight angle, and \( \overrightarrow{NQ} \) bisects \( \angle MNP \). Draw \( \angle MNP \) and \( \overrightarrow{NQ} \). Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles.

### 1.4 Exercises

**Skill Practice**

1. **Vocabulary** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.

2. **Writing** Explain how to find the measure of \( \angle PQR \), shown at the right.

### Naming Angles and Angle Parts

In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.

3. \( \angle BAC \)

4. \( \angle NQT \)

5. \( \angle MTP \)
6. **NAMING ANGLES** Name three different angles in the diagram at the right.

**CLASSIFYING ANGLES** Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.

- $m \angle W = 180^\circ$
- $m \angle X = 30^\circ$
- $m \angle Y = 90^\circ$
- $m \angle Z = 95^\circ$

**MEASURING ANGLES** Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.

- $\angle JFL$
- $\angle GFH$
- $\angle GFK$
- $\angle GFL$

**NAMING AND CLASSIFYING** Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.

- $\angle ACB$
- $\angle ABC$
- $\angle BFD$
- $\angle AEC$
- $\angle BDC$
- $\angle Bec$

**TAKS REASONING** Which is a correct name for the obtuse angle in the diagram?

- A) $\angle ACB$
- B) $\angle ACD$
- C) $\angle BCD$
- D) $\angle C$

**ANGLE ADDITION POSTULATE** Find the indicated angle measure.

- $m \angle QST = ?$
- $m \angle ADC = ?$
- $m \angle NPM = ?$

**ALGEBRA** Use the given information to find the indicated angle measure.

- Given $m \angle WXZ = 80^\circ$, find $m \angle YXZ$.
- Given $m \angle FJH = 168^\circ$, find $m \angle FJG$.

**TAKS REASONING** In the diagram, the measure of $\angle XYZ$ is 140°. What is the value of $x$?

- A) 27
- B) 33
- C) 67
- D) 73
28. **CONGRUENT ANGLES** In the photograph below, \( m\angle AED = 34^\circ \) and \( m\angle EAD = 112^\circ \). Identify the congruent angles in the diagram. Then find \( m\angle BDC \) and \( m\angle ADB \).

![Diagram of a pattern with labeled angles](image)

29. **ANGLE BISECTORS** Given that \( \overrightarrow{WZ} \) bisects \( \angle XWY \), find the two angle measures not given in the diagram.

\[ \angle XWY = 52^\circ \]
\[ \angle YWZ = 68^\circ \]

30. \[ \angle WYZ = 71^\circ \]

31. \[ \angle WZX = 30^\circ \]

32. **ERROR ANALYSIS** \( \overrightarrow{KM} \) bisects \( \angle JKL \) and \( m\angle JKM = 30^\circ \). Describe and correct the error made in stating that \( m\angle JKL = 15^\circ \). Draw a sketch to support your answer.

33. \( \alpha = 142^\circ \)

34. \( \beta = 53^\circ \)

35. \( \gamma = 56^\circ \)

36. \( \delta = 143^\circ \)

37. \( \epsilon = 107^\circ \)

38. \( \phi = 31^\circ \)

39. **ERROR ANALYSIS** A student states that \( \overrightarrow{AD} \) can bisect \( \angle AGC \). Describe and correct the student’s error. Draw a sketch to support your answer.

40. **ALGEBRA** In each diagram, \( \overrightarrow{BD} \) bisects \( \angle ABC \). Find \( m\angle ABC \).

\[ AB = (4x - 2)^\circ \]
\[ AC = (3x + 18)^\circ \]

41. \[ AB = (2x + 20)^\circ \]
\[ AC = (x + 17)^\circ \]

42. \[ AB = (x - 33)^\circ \]
\[ AC = (x + 2)^\circ \]

43. **TAKS REASONING** You are measuring \( \angle PQR \) with a protractor. When you line up \( QR \) with the 20° mark, \( 
\) lines up with the 80° mark. Then you move the protractor so that \( QR \) lines up with the 15° mark. What mark does \( \overrightarrow{QP} \) line up with? Explain.

44. **ALGEBRA** Plot the points in a coordinate plane and draw \( \angle ABC \). Classify the angle. Then give the coordinates of a point that lies in the interior of the angle.

- \( A(3, 3), B(0, 0), C(3, 0) \)
- \( A(-5, 4), B(1, 4), C(-2, -2) \)
- \( A(-5, 2), B(-2, -2), C(4, -3) \)
- \( A(-3, -1), B(2, 1), C(6, -2) \)
48. **Algebra** Let \((2x - 12)\) represent the measure of an acute angle. What are the possible values of \(x\)?

49. **Challenge** \(\overrightarrow{SQ}\) bisects \(\angle RST\), \(\overrightarrow{SP}\) bisects \(\angle RSQ\), and \(\overrightarrow{SV}\) bisects \(\angle RSP\). The measure of \(\angle VSP\) is \(17^\circ\). Find \(m \angle TSQ\). Explain.

50. **Finding Measures** In the diagram,
\[
m \angle AEB = \frac{1}{2} \cdot m \angle CED, \text{ and } \angle AED
\]
is a straight angle. Find \(m \angle AEB\) and \(m \angle CED\).

**Problem Solving**

51. **Sculpture** In the sculpture shown in the photograph, suppose the measure of \(\angle LMN\) is \(79^\circ\) and the measure of \(\angle PMN\) is \(47^\circ\). What is the measure of \(\angle LMP\)?

52. **Map** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of \(162^\circ\). Park Road intersects Sydney Street at an angle of \(87^\circ\). Find the angle at which Malcom Way intersects Park Road.

53. In the roof truss, \(\overrightarrow{BG}\) bisects \(\angle ABC\) and \(\angle DEF\), \(m \angle ABC = 112^\circ\), and \(\angle ABC \equiv \angle DEF\). Find the measure of the following angles.

\[
a. \ m \angle DEF \quad b. \ m \angle ABG \\
c. \ m \angle CBG \quad d. \ m \angle DEG
\]

54. In the roof truss, \(\overrightarrow{GB}\) bisects \(\angle DGF\). Find \(m \angle DGE\) and \(m \angle FGE\).

55. Name an example of each of the following types of angles: acute, obtuse, right, and straight.
GEOGRAPHY For the given location on the map, estimate the measure of \( \angle PSL \), where \( P \) is on the Prime Meridian (0° longitude), \( S \) is the South Pole, and \( L \) is the location of the indicated research station.

56. Macquarie Island 57. Dumont d’Urville 58. McMurdo
59. Mawson 60. Syowa 61. Vostok

62. **TAKS REASONING** In the flag shown, \( \angle AFE \) is a straight angle and \( FC \) bisects \( \angle AFE \) and \( \angle BFD \).
   a. Which angles are acute? obtuse? right?
   b. Identify the congruent angles.
   c. If \( m \angle AFB = 26^\circ \), find \( m \angle DFE \), \( m \angle BFC \), \( m \angle CFD \), \( m \angle AFC \), \( m \angle AFD \), and \( m \angle BFD \). Explain.

63. **CHALLENGE** Create a set of data that could be represented by the circle graph at the right. Explain your reasoning.

---

**Mixed Review for TAKS**

**TAKS PRACTICE** The equation \( y = 2.6x^2 - 3.4x + 1.2 \) shows the relationship between \( x \), the number of years since a company began business, and \( y \), the company’s profit in millions of dollars. What is the company’s profit after they are in business for 8 years? **TAKS Obj. 2**

A \$138 million \hspace{1cm} B \$140.4 million \hspace{1cm} C \$192.4 million \hspace{1cm} D \$194.8 million

**TAKS PRACTICE** A cylindrical salt shaker has a height of 7 centimeters and a diameter of 4 centimeters. Mike fills the salt shaker to 0.5 centimeter from the top. Which expression can be used to find the volume of salt in the salt shaker? **TAKS Obj. 8**

F \( \pi(2^2)(7 - 0.5) \) \hspace{1cm} G \( \pi(7 - 0.5)^2(2) \) \hspace{1cm} H \( \pi(4^2)(7 - 0.5) \) \hspace{1cm} J \( \pi(7 - 0.5)^2(4) \)
1.4 Copy and Bisect Segments and Angles

MATERIALS • compass • straightedge

QUESTION How can you copy and bisect segments and angles?

A construction is a geometric drawing that uses a limited set of tools, usually a compass and straightedge. You can use a compass and straightedge (a ruler without marks) to construct a segment that is congruent to a given segment, and an angle that is congruent to a given angle.

EXPLORE 1 Copy a segment

Use the following steps to construct a segment that is congruent to $\overline{AB}$.

**STEP 1** Draw a segment Use a straightedge to draw a segment longer than $\overline{AB}$. Label point $C$ on the new segment.

**STEP 2** Measure length Set your compass at the length of $\overline{AB}$.

**STEP 3** Copy length Place the compass at $C$. Mark point $D$ on the new segment. $\overline{CD} \cong \overline{AB}$.

EXPLORE 2 Bisect a segment

Use the following steps to construct a bisector of $\overline{AB}$ and to find the midpoint $M$ of $\overline{AB}$.

**STEP 1** Draw an arc Place the compass at $A$. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

**STEP 2** Draw a second arc Keep the same compass setting. Place the compass at $B$. Draw an arc. It should intersect the other arc at two points.

**STEP 3** Bisect segment Draw a segment through the two points of intersection. This segment bisects $\overline{AB}$ at $M$, the midpoint of $\overline{AB}$.
**Explore 3** Copy an angle

Use the following steps to construct an angle that is congruent to $\angle A$. In this construction, the radius of an arc is the distance from the point where the compass point rests (the center of the arc) to a point on the arc drawn by the compass.

**Step 1**
- **Draw a segment** Draw a segment. Label a point $D$ on the segment.

**Step 2**
- **Draw arcs** Draw an arc with center $A$. Using the same radius, draw an arc with center $D$.

**Step 3**
- **Draw arcs** Label $B$, $C$, and $E$. Draw an arc with radius $BC$ and center $E$. Label the intersection $F$.

**Step 4**
- **Draw a ray** Draw $\overrightarrow{DF}$.

$\angle EDF \cong \angle BAC$.

**Explore 4** Bisect an angle

Use the following steps to construct an angle bisector of $\angle A$.

**Step 1**
- **Draw an arc** Place the compass at $A$. Draw an arc that intersects both sides of the angle. Label the intersections $C$ and $B$.

**Step 2**
- **Draw arcs** Place the compass at $C$. Draw an arc. Then place the compass point at $B$. Using the same radius, draw another arc.

**Step 3**
- **Draw a ray** Label the intersection $G$. Use a straightedge to draw a ray through $A$ and $G$. $\overrightarrow{AG}$ bisects $\angle A$.

**Draw Conclusions** Use your observations to complete these exercises

1. Describe how you could use a compass and a straightedge to draw a segment that is twice as long as a given segment.

2. Draw an obtuse angle. Copy the angle using a compass and a straightedge. Then bisect the angle using a compass and straightedge.
1.5 Describe Angle Pair Relationships

You used angle postulates to measure and classify angles. 

Now You will use special angle relationships to find angle measures. 

So you can find measures in a building, as in Ex. 53.

Key Vocabulary
• complementary angles
• supplementary angles
• adjacent angles
• linear pair
• vertical angles

Two angles are **complementary angles** if the sum of their measures is $90^\circ$. Each angle is the complement of the other. Two angles are **supplementary angles** if the sum of their measures is $180^\circ$. Each angle is the supplement of the other. 

Complementary angles and supplementary angles can be **adjacent angles** or **nonadjacent angles**. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.

---

**Example 1** Identify complements and supplements

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

**Solution**

Because $32^\circ + 58^\circ = 90^\circ$, $\angle BAC$ and $\angle RST$ are complementary angles.

Because $122^\circ + 58^\circ = 180^\circ$, $\angle CAD$ and $\angle RST$ are supplementary angles.

Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent.

**Guided Practice** for Example 1

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.
EXAMPLE 2  Find measures of a complement and a supplement

a. Given that \( \angle 1 \) is a complement of \( \angle 2 \) and \( m\angle 1 = 68^\circ \), find \( m\angle 2 \).

b. Given that \( \angle 3 \) is a supplement of \( \angle 4 \) and \( m\angle 4 = 56^\circ \), find \( m\angle 3 \).

Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

\[
m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 68^\circ = 22^\circ
\]

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

\[
m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 56^\circ = 124^\circ
\]

EXAMPLE 3  Find angle measures

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find \( m\angle BCE \) and \( m\angle ECD \).

Solution

STEP 1  Use the fact that the sum of the measures of supplementary angles is \( 180^\circ \).

\[
m\angle BCE + m\angle ECD = 180^\circ
\]

Write equation.

\[
(4x + 8)^\circ + (x + 2)^\circ = 180^\circ
\]

Substitute.

\[
5x + 10 = 180
\]

Combine like terms.

\[
x = 34
\]

Divide each side by 5.

STEP 2  Evaluate the original expressions when \( x = 34 \).

\[
m\angle BCE = (4x + 8)^\circ = (4 \cdot 34 + 8)^\circ = 144^\circ
\]

\[
m\angle ECD = (x + 2)^\circ = (34 + 2)^\circ = 36^\circ
\]

The angle measures are \( 144^\circ \) and \( 36^\circ \).

GUIDED PRACTICE  for Examples 2 and 3

3. Given that \( \angle 1 \) is a complement of \( \angle 2 \) and \( m\angle 2 = 8^\circ \), find \( m\angle 1 \).

4. Given that \( \angle 3 \) is a supplement of \( \angle 4 \) and \( m\angle 3 = 117^\circ \), find \( m\angle 4 \).

5. \( \angle LMN \) and \( \angle PQR \) are complementary angles. Find the measures of the angles if \( m\angle LMN = (4x - 2)^\circ \) and \( m\angle PQR = (9x + 1)^\circ \).
**EXAMPLE 4** Identify angle pairs

Identify all of the linear pairs and all of the vertical angles in the figure at the right.

**Solution**

To find vertical angles, look for angles formed by intersecting lines.

- ∠1 and ∠5 are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

- ∠1 and ∠4 are a linear pair.
- ∠4 and ∠5 are also a linear pair.

**Example 5** Find angle measures in a linear pair

**Problem**

Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.

**Solution**

Let \(x^\circ\) be the measure of one angle. The measure of the other angle is \(5x^\circ\). Then use the fact that the angles of a linear pair are supplementary to write an equation.

\[ x^\circ + 5x^\circ = 180^\circ \]
Write an equation.

\[ 6x = 180 \]
Combine like terms.

\[ x = 30 \]
Divide each side by 6.

The measures of the angles are 30° and 5(30°) = 150°.
Chapter 1  Essentials of Geometry

1.5 EXERCISES

**SKILL PRACTICE**

1. **VOCABULARY** Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? *Explain.*

2. **WRITING** Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? *Explain.*

**IDENTIFYING ANGLES** Tell whether the indicated angles are adjacent.

3. \(\angle ABD\) and \(\angle DBC\)

4. \(\angle WXY\) and \(\angle XYZ\)

5. \(\angle LQM\) and \(\angle NQM\)

**IDENTIFYING ANGLES** Name a pair of complementary angles and a pair of supplementary angles.

6. \(\angle OT\) and \(\angle TR\)

7. \(\angle HJ\) and \(\angle KL\)
1.5 Describe Angle Pair Relationships

**COMPLEMENTARY ANGLES** \( \angle 1 \) and \( \angle 2 \) are complementary angles. Given the measure of \( \angle 1 \), find \( m \angle 2 \).

8. \( m \angle 1 = 45^\circ \) 9. \( m \angle 1 = 21^\circ \) 10. \( m \angle 1 = 89^\circ \) 11. \( m \angle 1 = 5^\circ \)

**SUPPLEMENTARY ANGLES** \( \angle 1 \) and \( \angle 2 \) are supplementary angles. Given the measure of \( \angle 1 \), find \( m \angle 2 \).

12. \( m \angle 1 = 60^\circ \) 13. \( m \angle 1 = 155^\circ \) 14. \( m \angle 1 = 130^\circ \) 15. \( m \angle 1 = 27^\circ \)

16. **TAKS REASONING** The arm of a crossing gate moves 37° from vertical. How many more degrees does the arm have to move so that it is horizontal?

A 37°  B 53°  C 90°  D 143°

**ALGEBRA** Find \( m \angle D E G \) and \( m \angle G E F \).

17. \( (18x - 9)^\circ \) and \( (4x + 13)^\circ \) 18. \( (7x - 3)^\circ \) and \( (12x - 7)^\circ \) 19. \( 6x^\circ \) and \( 4x^\circ \)

**IDENTIFYING ANGLE PAIRS** Use the diagram below. Tell whether the angles are vertical angles, a linear pair, or neither.

20. \( \angle 1 \) and \( \angle 4 \) 21. \( \angle 1 \) and \( \angle 2 \) 22. \( \angle 3 \) and \( \angle 5 \) 23. \( \angle 2 \) and \( \angle 3 \) 24. \( \angle 7, \angle 8, \) and \( \angle 9 \) 25. \( \angle 5 \) and \( \angle 6 \) 26. \( \angle 6 \) and \( \angle 7 \) 27. \( \angle 5 \) and \( \angle 9 \)

28. **ALGEBRA** Two angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle.

29. **ERROR ANALYSIS** Describe and correct the error made in finding the value of \( x \).

30. **TAKS REASONING** The measure of one angle is 24° greater than the measure of its complement. What are the measures of the angles?

A 24° and 66°  B 24° and 156°  C 33° and 57°  D 78° and 102°

**ALGEBRA** Find the values of \( x \) and \( y \).

31. \( (9x + 20)^\circ \) or \( 2y^\circ \) 32. \( (5y + 38)^\circ \) or \( (8x + 26)^\circ \) 33. \( 2y^\circ \) and \( (4x - 100)^\circ \)

\( (3y + 30)^\circ \) and \( (x + 5)^\circ \)
REASONING  Tell whether the statement is always, sometimes, or never true. Explain your reasoning.

34. An obtuse angle has a complement.
35. A straight angle has a complement.
36. An angle has a supplement.
37. The complement of an acute angle is an acute angle.
38. The supplement of an acute angle is an obtuse angle.

FINDING ANGLES  ∠A and ∠B are complementary. Find m∠A and m∠B.

39. m∠A = (3x + 2)°  40. m∠A = (15x + 3)°  41. m∠A = (11x + 24)°
m∠B = (x - 4)°  m∠B = (5x - 13)°  m∠B = (x + 18)°

FINDING ANGLES  ∠A and ∠B are supplementary. Find m∠A and m∠B.

42. m∠A = (8x + 100)°  43. m∠A = (2x - 20)°  44. m∠A = (6x + 72)°
m∠B = (2x + 50)°  m∠B = (3x + 5)°  m∠B = (2x + 28)°

45. CHALLENGE You are given that ∠GHJ is a complement of ∠RST and ∠RST is a supplement of ∠ABC. Let m∠GHJ be x°. What is the measure of ∠ABC? Explain your reasoning.

PROBLEM SOLVING

IDENTIFYING ANGLES  Tell whether the two angles shown are complementary, supplementary, or neither.

46. 47. 48.

ARCHITECTURE  The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example type of the indicated type of angle pair.

49. Supplementary angles  50. Vertical angles
51. Linear pair  52. Adjacent angles

53. TAKS REASONING  Use the photograph shown at the right. Given that ∠FGB and ∠BGC are supplementary angles, and m∠FGB = 120°, explain how to find the measure of the complement of ∠BGC.
54. **SHADOWS** The length of a shadow changes as the sun rises. In the diagram below, the length of $\overline{CB}$ is the length of a shadow. The end of the shadow is the vertex of $\angle ABC$, which is formed by the ground and the sun’s rays. Describe how the shadow and angle change as the sun rises.

55. **MULTIPLE REPRESENTATIONS** Let $x^\circ$ be an angle measure. Let $y_1^\circ$ be the measure of a complement of the angle and let $y_2^\circ$ be the measure of a supplement of the angle.

   a. **Writing an Equation** Write equations for $y_1$ as a function of $x$, and for $y_2$ as a function of $x$. What is the domain of each function? Explain.

   b. **Drawing a Graph** Graph each function and describe its range.

56. **CHALLENGE** The sum of the measures of two complementary angles exceeds the difference of their measures by $86^\circ$. Find the measure of each angle. Explain how you found the angle measures.

---

**EXTRA PRACTICE** for Lesson 1.5, p. 897  
**ONLINE QUIZ** at classzone.com

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57. **TAXS PRACTICE** The point $(-1, y)$ is a solution of the equation $6x + 5y = 19$. What is the value of $y$? **TAXS Obj. 4**

   A  3  B  1  C  4  D  5

58. **TAXS PRACTICE** Anna swims diagonal laps in the pool shown. About how many laps must she complete to swim 0.5 mile? **TAXS Obj. 8**

   F  73  G  98  H  127  J  197

---

**QUIZ for Lessons 1.4–1.5**

In each diagram, $\overline{BD}$ bisects $\angle ABC$. Find $m\angle ABD$ and $m\angle DBC$. (p. 24)

1. \[ (x + 20)^\circ \quad (3x - 4)^\circ \]
2. \[ (10x - 42)^\circ \quad (6x + 10)^\circ \]
3. \[ (18x + 27)^\circ \quad (9x + 36)^\circ \]

Find the measure of (a) the complement and (b) the supplement of $\angle 1$. (p. 35)

4. $m\angle 1 = 47^\circ$  
5. $m\angle 1 = 19^\circ$  
6. $m\angle 1 = 75^\circ$  
7. $m\angle 1 = 2^\circ$
1.6 Classify Polygons

**Key Vocabulary**
- polygon
- side, vertex
- convex
- concave
- n-gon
- equilateral
- equiangular
- regular

**Before**
You classified angles.

**Now**
You will classify polygons.

**Why?**
So you can find lengths in a floor plan, as in Ex. 32.

**KEY CONCEPT**

**Identifying Polygons**
In geometry, a figure that lies in a plane is called a plane figure. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is **vertices**. A polygon can be named by listing the vertices in consecutive order. For example, ABCDE and CDEAB are both correct names for the polygon at the right.

A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called **nonconvex** or **concave**.

**Example 1**
Identify polygons

Tell whether the figure is a polygon and whether it is **convex** or **concave**.

**Solution**

a. Some segments intersect more than two segments, so it is not a polygon.

b. The figure is a convex polygon.

c. Part of the figure is not a segment, so it is not a polygon.

d. The figure is a concave polygon.

**READ VOCABULARY**
A plane figure is two-dimensional. Later, you will study three-dimensional space figures such as prisms and cylinders.
**CLASSIFYING POLYGONS** A polygon is named by the number of its sides.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Type of polygon</th>
<th>Number of sides</th>
<th>Type of polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
<td>8</td>
<td>Octagon</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
<td>9</td>
<td>Nonagon</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
<td>10</td>
<td>Decagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
<td>12</td>
<td>Dodecagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>n-gon</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The term \(\text{n-gon}\), where \(n\) is the number of a polygon’s sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent.
In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.

---

**EXAMPLE 2 Classify polygons**

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

**Solution**

a. The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.

b. The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.

c. The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

---

**GUIDED PRACTICE for Examples 1 and 2**

1. Sketch an example of a convex heptagon and an example of a concave heptagon.

2. Classify the polygon shown at the right by the number of sides. **Explain** how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.
GUIDED PRACTICE for Example 3

3. The expressions $8y^\circ$ and $(9y - 15)^\circ$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.

**Example 3** Find side lengths

**ALGEBRA** A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.

**Solution**

First, write and solve an equation to find the value of $x$. Use the fact that the sides of a regular hexagon are congruent.

\[
3x + 6 = 4x - 2 \quad \text{Write equation.}
\]

\[
6 = x - 2 \quad \text{Subtract } 3x \text{ from each side.}
\]

\[
8 = x \quad \text{Add } 2 \text{ to each side.}
\]

Then find a side length. Evaluate one of the expressions when $x = 8$.

\[
3x + 6 = 3(8) + 6 = 30
\]

The length of a side of the table is 30 inches.

1.6 EXERCISES

1. **VOCABULARY** Explain what is meant by the term $n$-gon.

2. **WRITING** Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? Explain.

3. **IDENTIFYING POLYGONS** Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is convex or concave.

4. **TAKS REASONING** Which of the figures is a concave polygon?

[Diagram of figures A, B, C, D]
**EXAMPLE 2** on p. 43 for Exs. 8–14

**CLASSIFYING** Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. **Explain** your reasoning.

8.  
9.  
10.  
11.  
12.  
13.  

14. **ERROR ANALYSIS** Two students were asked to draw a regular hexagon, as shown below. **Describe** the error made by each student.

**EXAMPLE 3** on p. 44 for Exs. 15–17

15. **ALGEBRA** The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. **ALGEBRA** The expressions $(9x + 5)^\circ$ and $(11x - 25)^\circ$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. **ALGEBRA** The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

**USING PROPERTIES** Tell whether the statement is **always**, **sometimes**, or **never** true.

18. A triangle is convex.  
19. A decagon is regular.  
20. A regular polygon is equiangular.  
21. A circle is a polygon.  
22. A polygon is a plane figure.  
23. A concave polygon is regular.

**DRAWING** Draw a figure that fits the description.

24. A triangle that is not regular  
25. A concave quadrilateral  
26. A pentagon that is equilateral but not equiangular  
27. An octagon that is equiangular but not equilateral

**ALGEBRA** Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of $x$.

28.  
29.  
30.  

1.6 Classify Polygons 45
31. **CHALLENGE** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.

32. **ARCHITECTURE** Longwood House, shown in the photograph on page 42, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.
   a. Tell whether the red polygon in the diagram is *convex* or *concave*.
   b. Classify the red polygon and tell whether it appears to be regular.

33. **SIGNS** Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral*, *equiangular*, or **regular**.
   34. ![Yield sign](image)
   35. ![Reserved parking sign](image)
   36. ![Stop sign](image)
   37. **TAKS REASONING** Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, −4)$. Which of the following could be the other two vertices?
      - A. $C(4, 4)$ and $D(4, −4)$
      - B. $C(−4, 4)$ and $D(−4, −4)$
      - C. $C(8, −4)$ and $D(8, 4)$
      - D. $C(0, 8)$ and $D(0, −8)$

38. **MULTI-STEP PROBLEM** The diagram shows the design of a lattice made in China in 1850.
   a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.
   b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.
39. **TAKS REASONING** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? Explain.

40. **TAKS REASONING** A segment that joins two nonconsecutive vertices of a polygon is called a *diagonal*. For example, a quadrilateral has two diagonals, as shown below.

<table>
<thead>
<tr>
<th>Type of polygon</th>
<th>Diagram</th>
<th>Number of sides</th>
<th>Number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrilateral</td>
<td></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Pentagon</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Hexagon</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Heptagon</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

a. Copy and complete the table. Describe any patterns you see.
b. How many diagonals does an octagon have? a nonagon? Explain.
c. The expression \( \frac{n(n - 3)}{2} \) can be used to find the number of diagonals in an \( n \)-gon. Find the number of diagonals in a 60-gon.

41. **LINE SYMMETRY** A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

a. A regular triangle  
b. A regular pentagon  
c. A regular hexagon  
d. A regular octagon

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge.

**Mixed Review for TAKS**

42. **TAKS PRACTICE** A function \( f(x) = 2x^2 + 1 \) has \{1, 3, 5, 8\} as the replacement set for the independent variable \( x \). Which of the following is contained in the corresponding set for the dependent variable? TAKS Obj. 1

- A 0  
- B 5  
- C 15  
- D 19

43. **TAKS PRACTICE** The radius of Cylinder A is three times the radius of Cylinder B. The heights of the cylinders are equal. How many times greater is the volume of Cylinder A than the volume of Cylinder B? TAKS Obj. 8

- F 3  
- G 6  
- H 9  
- J 27
1.7 Investigate Perimeter and Area

**MATERIALS** • graph paper • graphing calculator

**QUESTION** How can you use a graphing calculator to find the smallest possible perimeter for a rectangle with a given area?

You can use the formulas below to find the perimeter $P$ and the area $A$ of a rectangle with length $l$ and width $w$.

$$P = 2l + 2w \quad \text{and} \quad A = lw$$

**EXPLORE** Find perimeters of rectangles with fixed areas

**STEP 1** Draw rectangles

Draw different rectangles, each with an area of 36 square units. Use lengths of 2, 4, 6, 8, 10, 12, 14, 16, and 18 units.

**STEP 2** Enter data

Use the STATISTICS menu on a graphing calculator. Enter the rectangle lengths in List 1. Use the keystrokes below to calculate and enter the rectangle widths and perimeters in Lists 2 and 3.

Keystrokes for entering widths in List 2:

```
36 + 2nd [L1] ENTER
```

Keystrokes for entering perimeters in List 3:

```
2 × 2nd [L1] ÷ 2nd 2 × [L2] ENTER
```

**STEP 3** Make a scatter plot

Make a scatter plot using the lengths from List 1 as the $x$-values and the perimeters from List 3 as the $y$-values. Choose an appropriate viewing window. Then use the trace feature to see the coordinates of each point.

How does the graph show which of your rectangles from Step 1 has the smallest perimeter?

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. Repeat the steps above for rectangles with areas of 64 square units.

2. Based on the Explore and your results from Exercise 1, what do you notice about the shape of the rectangle with the smallest perimeter?
1.7 Find Perimeter, Circumference, and Area

Key Vocabulary
- perimeter, p. 923
- circumference, p. 923
- area, p. 923
- diameter, p. 923
- radius, p. 923

Recall that **perimeter** is the distance around a figure, **circumference** is the distance around a circle, and **area** is the amount of surface covered by a figure. Perimeter and circumference are measured in units of length, such as meters (m) and feet (ft). Area is measured in square units, such as square meters (m²) and square feet (ft²).

**KEY CONCEPT**

### Formulas for Perimeter P, Area A, and Circumference C

#### Square
- side length s
  - Perimeter: \( P = 4s \)
  - Area: \( A = s^2 \)

#### Rectangle
- length \( l \) and width \( w \)
  - Perimeter: \( P = 2l + 2w \)
  - Area: \( A = lw \)

#### Triangle
- side lengths \( a \), \( b \), and \( c \), base \( b \), and height \( h \)
  - Perimeter: \( P = a + b + c \)
  - Area: \( A = \frac{1}{2}bh \)

#### Circle
- diameter \( d \) and radius \( r \)
  - Circumference: \( C = \pi d = 2\pi r \)
  - Area: \( A = \pi r^2 \)

Pi (\( \pi \)) is the ratio of a circle's circumference to its diameter.

### Example 1

**Find the perimeter and area of a rectangle**

**BASKETBALL** Find the perimeter and area of the rectangular basketball court shown.

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P = 2l + 2w )</td>
<td>( A = lw )</td>
</tr>
<tr>
<td>( = 2(84) + 2(50) )</td>
<td>( = 84(50) )</td>
</tr>
<tr>
<td>( = 268 )</td>
<td>( = 4200 )</td>
</tr>
</tbody>
</table>

The perimeter is 268 feet and the area is 4200 square feet.
EXAMPLE 2 Find the circumference and area of a circle

TEAM PATCH You are ordering circular cloth patches for your soccer team’s uniforms. Find the approximate circumference and area of the patch shown.

Solution
First find the radius. The diameter is 9 centimeters, so the radius is \( \frac{1}{2}(9) = 4.5 \) centimeters.
Then find the circumference and area. Use 3.14 to approximate the value of \( \pi \).

\[
C = 2\pi r = 2(3.14)(4.5) = 28.26 \\
A = \pi r^2 = 3.14(4.5)^2 = 63.585
\]

The circumference is about 28.3 cm. The area is about 63.6 cm\(^2\).

GUIDED PRACTICE for Examples 1 and 2
Find the area and perimeter (or circumference) of the figure. If necessary, round to the nearest tenth.

1. \(13 \text{ m} \quad 5.7 \text{ m} \)
2. \(1.6 \text{ cm} \)
3. \(2 \text{ yd} \)

EXAMPLE 3 TAKS PRACTICE: Multiple Choice

\( \triangle QRS \) has vertices at \( Q(2, 1) \), \( R(3, 6) \), and \( S(6, 1) \). What is the approximate perimeter of \( \triangle QRS \)?

\( \text{(A) 8 units \quad (B) 8.7 units \quad (C) 14.9 units \quad (D) 29.8 units} \)

Solution
First draw \( \triangle QRS \) in a coordinate plane. Then find the side lengths. Because \( QS \) is horizontal, find \( QS \) by using the Ruler Postulate. Use the distance formula to find \( QR \) and \( SR \).

\[
QS = |6 - 2| = 4 \text{ units} \\
QR = \sqrt{(3 - 2)^2 + (6 - 1)^2} = \sqrt{26} \approx 5.1 \text{ units} \\
RS = \sqrt{(6 - 3)^2 + (1 - 6)^2} = \sqrt{34} \approx 5.8 \text{ units}
\]

Find the perimeter.

\[
P = QS + QR + SR \approx 4 + 5.1 + 5.8 = 14.9 \text{ units}
\]

The correct answer is C. (A) (B) (C) (D)
EXAMPLE 4 Solve a multi-step problem

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute.

About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

Solution

The machine can resurface the ice at a rate of 270 square yards per minute. So, the amount of time it takes to resurface the skating rink depends on its area.

**STEP 1** Find the area of the rectangular skating rink.

\[ \text{Area} = lw = 200(90) = 18,000 \text{ ft}^2 \]

The resurfacing rate is in square yards per minute. Rewrite the area of the rink in square yards. There are 3 feet in 1 yard, and \( 3^2 = 9 \) square feet in 1 square yard.

\[ 18,000 \text{ ft}^2 \div \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 2000 \text{ yd}^2 \]

Use unit analysis.

**STEP 2** Write a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let \( t \) represent the total time (in minutes) needed to resurface the skating rink.

\[
\begin{align*}
\text{Area of rink} & \quad \text{(yd}^2) \\
\text{Resurfacing rate} & \quad \text{(yd}^2 \text{ per min)} \\
\text{Total time} & \quad \text{(min)} \\
\hline
2000 & = 270 \cdot t \\
7.4 & \approx t
\end{align*}
\]

It takes the ice-resurfacing machine about 7 minutes to resurface the skating rink.

**GUIDED PRACTICE**

for Examples 3 and 4

4. Describe how to find the height from \( F \) to \( \overline{EG} \) in the triangle at the right.

5. Find the perimeter and the area of the triangle shown at the right.

6. WHAT IF? In Example 4, suppose the skating rink is twice as long and twice as wide. Will it take an ice-resurfacing machine twice as long to resurface the skating rink? Explain your reasoning.
Example 5  Find unknown length

The base of a triangle is 28 meters. Its area is 308 square meters. Find the height of the triangle.

Solution

\[ A = \frac{1}{2}bh \]  
Write formula for the area of a triangle.
\[ 308 = \frac{1}{2}(28)h \]  
Substitute 308 for \( A \) and 28 for \( b \).
\[ 22 = h \]  
Solve for \( h \).

The height is 22 meters.

Guided Practice for Example 5

7. The area of a triangle is 64 square meters, and its height is 16 meters. Find the length of its base.

1. Vocabulary  How are the diameter and radius of a circle related?

2. Writing  Describe a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

3. Error Analysis  Describe and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet.

\[ A = 52(9) = 468 \text{ ft}^2 \]

Perimeter and Area  Find the perimeter and area of the shaded figure.

4. 5. 6.

7. 8. 9.

1.7 Exercises

Skill Practice

1. Vocabulary  How are the diameter and radius of a circle related?

2. Writing  Describe a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

3. Error Analysis  Describe and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet. 

\[ A = 52(9) = 468 \text{ ft}^2 \]

Perimeter and Area  Find the perimeter and area of the shaded figure.
10. **DRAWING A DIAGRAM** The base of a triangle is 32 feet. Its height is \(16\frac{1}{2}\) feet. Sketch the triangle and find its area.

**CIRCUMFERENCE AND AREA** Use the given diameter \(d\) or radius \(r\) to find the circumference and area of the circle. Round to the nearest tenth.

11. \(d = 27\) cm
12. \(d = 5\) in.
13. \(r = 12.1\) cm
14. \(r = 3.9\) cm

15. **DRAWING A DIAGRAM** The diameter of a circle is 18.9 centimeters. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

**DISTANCE FORMULA** Find the perimeter of the figure. Round to the nearest tenth of a unit.

16.

17.

18.

19. **TAKS REASONING** What is the approximate area (in square units) of the rectangle shown at the right?

- A 6.7
- B 8.0
- C 9.0
- D 10.0

**CONVERTING UNITS** Copy and complete the statement.

20. \(187\) cm\(^2\) = ? m\(^2\)
21. \(13\) ft\(^2\) = ? yd\(^2\)
22. \(18\) in.\(^2\) = ? ft\(^2\)
23. \(8\) km\(^2\) = ? m\(^2\)
24. \(12\) yd\(^2\) = ? ft\(^2\)
25. \(24\) ft\(^2\) = ? in.\(^2\)

26. **TAKS REASONING** A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?

- A 27 in.\(^2\)
- B 54 in.\(^2\)
- C 144 in.\(^2\)
- D 324 in.\(^2\)

**UNKNOWN MEASURES** Use the information about the figure to find the indicated measure.

27. Area = 261 m\(^2\)
Find the height \(h\).

28. Area = 66 in.\(^2\)
Find the base \(b\).

29. Perimeter = 25 in.
Find the width \(w\).
30. **UNKNOWN MEASURE** The width of a rectangle is 17 inches. Its perimeter is 102 inches. Find the length of the rectangle.

31. **ALGEBRA** The area of a rectangle is 18 square inches. The length of the rectangle is twice its width. Find the length and width of the rectangle.

32. **ALGEBRA** The area of a triangle is 27 square feet. Its height is three times the length of its base. Find the height and base of the triangle.

33. **ALGEBRA** Let \( x \) represent the side length of a square. Find a regular polygon with side length \( x \) whose perimeter is twice the perimeter of the square. Find a regular polygon with side length \( x \) whose perimeter is three times the length of the square. *Explain* your thinking.

**FINDING SIDE LENGTHS** Find the side length of the square with the given area. Write your answer as a radical in simplest form.

34. \( A = 184 \text{ cm}^2 \)  
35. \( A = 346 \text{ in.}^2 \)  
36. \( A = 1008 \text{ mi}^2 \)  
37. \( A = 1050 \text{ km}^2 \)

38. **TAKS REASONING** In the diagram, the diameter of the yellow circle is half the diameter of the red circle. What fraction of the area of the red circle is not covered by the yellow circle? *Explain*.

39. **CHALLENGE** The area of a rectangle is 30 \( \text{cm}^2 \) and its perimeter is 26 cm. Find the length and width of the rectangle.

---

**PROBLEM SOLVING**

**EXAMPLES**

1 and 2 on pp. 49–50 for Exs. 40–41

**EXAMPLE 4** on p. 51 for Ex. 42

---

40. **WATER LILIES** The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answers to the nearest tenth.

41. **LAND** You are planting grass on a rectangular plot of land. You are also building a fence around the edge of the plot. The plot is 45 yards long and 30 yards wide. How much area do you need to cover with grass seed? How many feet of fencing do you need?

42. **MULTI-STEP PROBLEM** Chris is installing a solar panel. The maximum amount of power the solar panel can generate in a day depends in part on its area. On a sunny day in the city where Chris lives, each square meter of the panel can generate up to 125 watts of power. The flat rectangular panel is 84 centimeters long and 54 centimeters wide.

   a. Find the area of the solar panel in square meters.
   
   b. What is the maximum amount of power (in watts) that the panel could generate if its area was 1 square meter? 2 square meters? *Explain*.
   
   c. Estimate the maximum amount of power Chris’s solar panel can generate. *Explain* your reasoning.
43. **MULTI-STEP PROBLEM** The eight spokes of a ship’s wheel are joined at the wheel’s center and pass through a large wooden circle, forming handles on the outside of the circle. From the wheel’s center to the tip of the handle, each spoke is 21 inches long.
   a. The circumference of the outer edge of the large wooden circle is 94 inches. Find the radius of the outer edge of the circle to the nearest inch.
   b. Find the length \( x \) of a handle on the wheel. *Explain.*

44. **MULTIPLE REPRESENTATIONS** Let \( x \) represent the length of a side of a square. Let \( y_1 \) and \( y_2 \) represent the perimeter and area of that square.
   a. **Making a Table** Copy and complete the table.

<table>
<thead>
<tr>
<th>Length, ( x )</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter, ( y_1 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Area, ( y_2 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

   b. **Making a Graph** Use the completed table to write two sets of ordered pairs: \((x, y_1)\) and \((x, y_2)\). Graph each set of ordered pairs.
   c. **Analyzing Data** Describe any patterns you see in the table from part (a) and in the graphs from part (b).

45. **TAKS REASONING** The photograph at the right shows the Crown Fountain in Chicago, Illinois. At this fountain, images of faces appear on a large screen. The images are created by light-emitting diodes (LEDs) that are clustered in groups called modules.
   The LED modules are arranged in a rectangular grid.
   a. The rectangular grid is approximately 7 meters wide and 15.2 meters high. Find the area of the grid.
   b. Suppose an LED module is a square with a side length of 4 centimeters. How many rows and how many columns of LED modules would be needed to make the Crown Fountain screen? *Explain* your reasoning.

46. **ASTRONOMY** The diagram shows a gap in Saturn’s circular rings. This gap is known as the Cassini division. In the diagram, the red circle represents the ring that borders the inside of the Cassini division. The yellow circle represents the ring that borders the outside of the division.
   a. The radius of the red ring is 115,800 kilometers. The radius of the yellow ring is 120,600 kilometers. Find the circumference of the red ring and the circumference of the yellow ring. Round your answers to the nearest hundred kilometers.
   b. Compare the circumferences of the two rings. About how many kilometers greater is the yellow ring’s circumference than the red ring’s circumference?
47. **CHALLENGE** In the diagram at the right, how many times as great is the area of the circle as the area of the square? *Explain* your reasoning.

48. **ALGEBRA** You have 30 yards of fencing with which to make a rectangular pen. Let *x* be the length of the pen.
   a. Write an expression for the width of the pen in terms of *x*. Then write a formula for the area *y* of the pen in terms of *x*.
   b. You want the pen to have the greatest possible area. What length and width should you use? *Explain* your reasoning.

---

**Mixed Review for TAKS**

49. **TAKS PRACTICE** Alexis is covering a Styrofoam ball with moss for a science fair project. She knows the radius of the ball and the number of square feet that one bag of moss will cover. Which formula should she use to determine the number of bags of moss needed? **TAKS Obj. 7**
   - A) \( V = \frac{4}{3} \pi r^3 \)
   - B) \( V = \frac{1}{3} \pi r^2 h \)
   - C) \( S = 4 \pi r^2 \)
   - D) \( S = 4 \pi rh \)

50. **TAKS PRACTICE** What are the coordinates of *A* after the translation \((x, y) \rightarrow (x - 1, y + 2)\)? **TAKS Obj. 6**
   - F) \((0, 0)\)
   - G) \((1, -3)\)
   - H) \((-2, 0)\)
   - J) \((-2, -4)\)

---

**QUIZ for Lessons 1.6–1.7**

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is **convex** or **concave**. *(p. 42)*

1. 
2. 
3. 

Find the perimeter and area of the shaded figure. *(p. 49)*

4. 
5. 
6. 

7. **GARDENING** You are spreading wood chips on a rectangular garden. The garden is \(3 \frac{1}{2}\) yards long and \(2 \frac{1}{2}\) yards wide. One bag of wood chips covers 10 square feet. How many bags of wood chips do you need? *(p. 49)*
Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 51

MULTIPLE REPRESENTATIONS In Example 4 on page 51, you saw how to use an equation to solve a problem about a skating rink. Looking for a pattern can help you write an equation.

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute. About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

Using a Pattern You can use a table to look for a pattern.

STEP 1 Find the area of the rink in square yards. In Example 4 on page 51, you found that the area was 2000 square yards.

STEP 2 Make a table that shows the relationship between the time spent resurfacing the ice and the area resurfaced. Look for a pattern.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Area resurfaced (yd²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 ⋅ 270 = 270</td>
</tr>
<tr>
<td>2</td>
<td>2 ⋅ 270 = 540</td>
</tr>
<tr>
<td>t</td>
<td>t ⋅ 270 = A</td>
</tr>
</tbody>
</table>

STEP 3 Use the equation to find the time \( t \) (in minutes) that it takes the machine to resurface 2000 square yards of ice.

\[
270t = A \\
270t = 2000 \\
t \approx 7.4
\]

It takes about 7 minutes.

PRACTICE

1. **PLOWING** A square field is \( \frac{1}{8} \) mile long on each side. A tractor can plow about 180,000 square feet per hour. To the nearest tenth of an hour, about how long does it take to plow the field? (1 mi = 5280 ft.)

2. **ERROR ANALYSIS** To solve Exercise 1 above, a student writes the equation \( 660 = 180,000t \), where \( t \) is the number of hours spent plowing. Describe and correct the error in the equation.

3. **PARKING LOT** A rectangular parking lot is 110 yards long and 45 yards wide. It costs about \$60 to pave each square foot of the parking lot with asphalt. About how much will it cost to pave the parking lot?

4. **WALKING** A circular path has a diameter of 120 meters. Your average walking speed is 4 kilometers per hour. About how many minutes will it take you to walk around the path 3 times?
MULTIPLE CHOICE

1. ROOFING Jane is covering the roof of a shed with shingles. The roof is a rectangle that is 4 yards long and 3 yards wide. Asphalt shingles cost $.75 per square foot and wood shingles cost $1.15 per square foot. How much more would Jane pay to use wood shingles instead of asphalt shingles? TEKS G.8.A

A $4.80  B $14.40  C $43.20  D $50.09

2. SNOWFLAKE The snowflake in the photo below can be circumscribed by a hexagon. Which of the following figures found in the hexagon is a concave polygon? TEKS G.9.B

F Triangle ABG  G Quadrilateral ABEF  H Hexagon ABCDEG  J Hexagon ABCDEF

3. DOOR FRAME The diagram shows a carving on a door frame. ∠HGD and ∠HGF are right angles, m∠DGB = 21°, m∠HBG = 55°, ∠DGB ≅ ∠CGF, and ∠HBG ≅ ∠HCG. What is m∠HGC? TEKS G.4

A 21°  B 69°  C 111°  D 159°

4. GARDEN Jim wants to lay bricks end-to-end around the border of the garden as shown below. Each brick is 10 inches long. Which expression can be used to find the number of bricks Jim needs? TEKS G.8.B

\[ \frac{26 \pi}{10} \]  \[ \frac{52 \pi}{12} \]  \[ 13\sqrt{2} \cdot \frac{12}{10} \]  \[ 13 \pi \cdot \frac{10}{12} \]

5. AREA The points A(−4,0), B(0,2), C(4,0), and D(0,−2) are plotted on a coordinate grid to form the vertices of a quadrilateral. What is the area of quadrilateral ABCD? TEKS G.8.A

F 8 square units  G 16 square units  H 20 square units  J 32 square units

6. ANGLES ∠1 and ∠2 are supplementary angles, and ∠1 and ∠3 are complementary angles. If m∠1 is 28° less than m∠2, what is m∠3 in degrees? TEKS G.4

A 21°  B 69°  C 111°  D 159°

7. SKATEBOARDING As shown in the diagram, a skateboarder tilts one end of a skateboard. What is m∠ZWX in degrees? TEKS G.5.A

A (2x + 5)°  B (9x - 1)°  C 2x + 5  D 9x - 1

58 Chapter 1 Essentials of Geometry
Describing Geometric Figures
You learned to identify and classify geometric figures.

<table>
<thead>
<tr>
<th>Point A</th>
<th>Line $\overline{AB}$</th>
<th>Plane $M$</th>
<th>Segment $\overline{AB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\overline{AB}$</td>
<td>$M$</td>
<td>$\overline{AB}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ray $\overline{AB}$</th>
<th>Angle $\angle A$</th>
<th>Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{AB}$</td>
<td>$\angle A$</td>
<td>Quadrilateral $ABCD$</td>
</tr>
</tbody>
</table>

Measuring Geometric Figures

**SEGMENTS** You measured segments in the coordinate plane.

**Distance Formula**
Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$:
$$ AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} $$

**Midpoint Formula**
Coordinates of midpoint $M$ of $\overline{AB}$, with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$:
$$ M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) $$

**ANGLES** You classified angles and found their measures.

**Complementary angles**
$$ m\angle 1 + m\angle 2 = 90^\circ $$

**Supplementary angles**
$$ m\angle 3 + m\angle 4 = 180^\circ $$

**FORMULAS** Perimeter and area formulas are reviewed on page 49.

Understanding Equality and Congruence
Congruent segments have equal lengths. Congruent angles have equal measures.

$\overline{AB} = \overline{BC}$ and $\overline{AB} = \overline{BC}$

$\angle JKL = \angle LKM$ and $m\angle JKL = m\angle LKM$
1. Copy and complete: Points $A$ and $B$ are the _?_ of $AB$.

2. Draw an example of a _linear pair_.

3. If $Q$ is between points $P$ and $R$ on $\overrightarrow{PR}$, and $PQ = QR$, then $Q$ is the _?_ of $PR$.

**REVIEW EXAMPLES AND EXERCISES**

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 1.

**Identify Points, Lines, and Planes**

**Example**

Use the diagram shown at the right.

Another name for $\overrightarrow{CD}$ is line $m$.

Points $A$, $B$, and $C$ are collinear.

Points $A$, $B$, $C$, and $F$ are coplanar.

**Exercises**

4. Give another name for line $g$.

5. Name three points that are _not_ collinear.

6. Name four points that are coplanar.

7. Name a pair of opposite rays.

8. Name the intersection of line $h$ and plane $M$.